

Surface Action for a Point Particle

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A remark on the movement of a point mass particle is given. If one associates to the particle a sphere of radius equal to the related De Broglie length, the relativistic action on the trajectory is proportional to the surface described by this sphere.

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1. Introduction

In standard relativistic mechanics, the base space-time is considered to be the Minkowski flat space, i.e. a real 4-dimensional manifold endowed with the Lorentzian metric η_{ik} ¹. A common procedure to derive the general properties of the movement is the use of the principle of least action. For a point particle of rest mass m_0 , the action S may be written¹:

$$S = -m_0 c \int_i^f (\eta_{ik} dx^i dx^k)^{1/2} \equiv -m_0 c \int_i^f ds \sim L_{if} \quad (1)$$

as being proportional to the *length* L_{if} of the world line of the point particle in the Minkowski space between some initial i and final f points; ds is the element of length of the world line, c is the speed of light and v is the speed of the particle. In string theories, the particle becomes a 1-dimensional object. The simplest type of action is the Nambu-Goto one²:

$$S = T \int_i^f [\dot{X}^2 X'^2 - (\dot{X} \cdot X')^2]^{1/2} d\sigma d\tau \equiv T \int_i^f d\Sigma \sim \Sigma_{if} \quad (2)$$

which is proportional to the *area* Σ_{if} of the world sheet of the string embedded in the Minkowski space; $d\Sigma$ is the element of area of the world sheet and T is a constant of proportionality introduced in order to make the expression of the dimensions of action. No more details are needed for the further understanding of the paper. If necessary, see².

The conclusions of this part is that the dynamic properties of the movement of the particles may be described by means of an action which is proportional to geometrical objects attached to the Minkowski space-time description (the *length* for the point particle and the *area* for the string). These concepts does not change in fact when passing from classical to quantum mechanics.

2. Surface proportional action for a point particle

We shall study the movement of a point particle of mass m . In the wave quantum mechanics, one may associate to this particle the De Broglie characteristic length Λ_B ³:

$$\Lambda_B = \frac{h}{2\pi p} = \frac{h}{2\pi mv} \quad (3)$$

where h is the Planck action constant, p - the momentum, m - the mass and v the velocity of the particle in some inertial frame (let's say the laboratory frame).

From the quantum mechanical point of view, we may consider in a good approximation that the particle is confined in a spatial zone delimited by a sphere whose radius is just the De Broglie length Λ_B . In this approximation, we are allowed to introduce a sphere of radius Λ_B attached to the particle. If the particle moves onto the direction x , this sphere determines an axis symmetric surface around the x axis in the laboratory frame. The area of this surface between the positions x_i and x_f is, obviously:

$$A_{if} = 2\pi \int_{x_i}^{x_f} \Lambda_B \cdot dx = 2\pi \int_{x_i}^{x_f} \frac{h}{2\pi mv} \cdot dx = \int_{x_i}^{x_f} \frac{h}{mv} \cdot dx \quad (4)$$

By replacing $dx = v \cdot dt$ and taking into account the relativistic mass dependence on speed¹, A_B becomes:

$$A_{if} = \int_{t_i}^{t_f} \frac{h}{m} \cdot dt = \frac{h}{m_0} \int_{t_i}^{t_f} \left(1 - \frac{v^2}{c^2}\right)^{1/2} \cdot dt \quad (5)$$

where m_0 is the rest mass of the particle and c - the speed of light.

On the other side, the relativistic action 1 for the same point mass particle is:

$$S_{if} = -m_0 c \int_i^f ds = -m_0 c^2 \int_i^f \frac{ds}{c} = -m_0 c^2 \int_{t_i}^{t_f} \left(1 - \frac{v^2}{c^2}\right)^{1/2} \cdot dt \quad (6)$$

From 5 and 6, we are able to assume that the relativistic action for a particle of rest mass m_0 is:

$$S_{if} = \frac{m_0^2 c^2}{h} \cdot A_{if} = \frac{h}{\lambda^2} \cdot A_{if} \quad (7)$$

proportional to the area A_{if} of the spatial surface generated by the De Broglie length radius sphere; λ is the equivalent Compton length for the particle of rest mass m_0 .

3. Conclusions

There are two general distinct methods to describe a particle: point particle or string. The dynamics of a point particle is described by a world line length proportional action, meanwhile the string action is proportional to the area of the world sheet. Starting from basic quantum mechanical assumptions, we considered that a point particle is confined in the interior of a sphere of radius equal to the attached De Broglie wavelength. After simple calculations, we proved that the area of the spatial surface described by this sphere attached to the particle is proportional to the standard relativistic action. This toy model could establish an intuitive argument for the progress of peace in the particles vs. strings war.

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